Reflections on Teaching Discrete Math for the First Time

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Abstract

Teaching a class for the first time is always a challenge. Add to the new experience a team-teaching approach and a professor teaching outside of his comfort zone, and things get even more complicated. In this article we evaluate our team-teaching of Discrete Math to computer science majors at the United States Military Academy. We share the aspects of the course that we think were successful as well as those that we would alter the next time around. This article should help any instructor as he/she prepares to teach a new course, particularly one in discrete mathematics.

Keywords: Teaching, Team-teaching, Discrete Mathematics

1. Introduction

Last semester we taught our first class in Discrete Math. At the beginning of the semester we had many ideas (and concerns) about the course that lay ahead of us [1]. Neither of us had taught the course before. Rachelle is a professor in the Mathematical Sciences department and Kevin is a Computer Science professor at the United States Military Academy at West Point. In this follow-up article, we will discuss our original goals and what we actually implemented. We will then discuss the lessons learned and our plans for the future.

2. Course Goals

Our overarching goal for the course was to improve the mathematical reasoning of our Computer Science students. Since the discrete math course is the only math course that directly supports the CS majors, we wanted to use it to enhance the students' abilities to reason mathematically on CS problems. To accomplish this goal, we followed the ACM 2001 recommendations for a CS focused discrete math course. In particular, the topics we focused on included logic and proofs; sets, functions and sequences; induction and recursion; counting; and graph and trees. With each new topic introduced, we emphasized proof techniques relevant to the material and assigned more application problems rather than ones that required only recall. In addition to our course content goals, we had objectives for course delivery. In particular, we aimed to take a team-teaching approach

3. Implementation

A total of 20 third year cadets were enrolled in the course and these students were divided into two sections of ten cadets each. Each of our students had taken the standard 4 semesters of math (math modeling, differential/integral calculus, and probability and statistics). According to the official registration, each of us was assigned one section of the course.

We began each class session with three "quick questions" based on an idea we discussed with Peter Henderson of Butler University. These questions served as a review of some of the main concepts of the previous day's lesson. We sent the students to the boards to answer questions that we projected on the front board. The students had just a minute or so to come up with a solution. These quick questions served to let each student judge whether he/she had learned the basics of the previous lesson and we found them to be very effective. A sample of these questions from the various topics follows:

- Is the following true or false: The converse of a conditional statement and its contrapositive has the same truth values.
- Draw a Venn diagram of the following for sets A , $\boldsymbol{B}$ and $\boldsymbol{C}$: $(\boldsymbol{A} \subseteq \boldsymbol{B}) \wedge (\boldsymbol{A} \subseteq \boldsymbol{C})$.
- List all the integers less than 15 that are relatively prime to 15.
- How do you use the induction hypothesis to prove a formula for a summation?

- In a group of 47 students, how many can you guarantee are born in the same month? What principle supports your answer?
- Use a tree diagram to determine the number of ways playoff games can occur in a best out of three contest.

After our quick questions, we would pass out a note-taking handout and delve into the material of the day. Usually the lead instructor would be at the front of the room, writing main concepts on the board while the students contributed what they had learned from the reading the night before. Interspersed throughout the remainder of the lesson were problems for the students to work, again usually at the board. As the students worked on problems, they were encouraged to talk with each other and the instructors. We would ask different students to share their solutions to problems throughout the class. Sometimes an incorrect solution was the best to share because it allowed everyone to learn from common mistakes. The problems worked during class were usually fairly straightforward problems that allowed the students to work with new concepts and new mathematical objects so that they could complete more difficult and deeper problems for homework. These problems were often altered from the course text book *Discrete Mathematics and its Applications, 6th* edition by Kenneth H. Rosen or taken from other texts such as *Discrete Mathematics with Applications* by Susanna S. Epp. The following samples are representative of the kinds of problems we asked during class throughout the course:

- Use De Morgan's laws to negate the following: $-1 < x \leq 4$.
- Prove by contraposition that for all integers n, if n^2 is even then n is even.
- Prove by induction that $n! < n^n$ for all integers $n > 1$.
- How many different ways can you arrange the letters of the word ARMY?
- Find a recurrence relation satisfied by the sequence $a_n = n^2 + n$.
- What kind of a graph can be used to model a highway system between major cities where there is an edge between the vertices representing cities for each interstate highway between them?
- Devise an algorithm similar to Kruskal's algorithm for constructing a maximum spanning tree of a connected weighted graph.

While we thought most class days were successful, two particular class days we found to be great learning days for our students were the day before our second midterm exam and the last day of class. The students were asked to bring review problems to class on the day before our second exam. We instructed them to bring "exam-worthy" problems to class and on a separate sheet of paper to write the solution. When they got to class we had them pass each problem to the student seated to their right who now had to solve the problem as best he/she could. Once the solver was finished with the problem, he/she handed it back to the problem writer who then "graded" it. This exercise gave the students a greater appreciation for how difficult it is to write good problems and write a complete and thorough solution. Also, in writing the problems, they had to decide which concepts were most important in order to provide a good review for themselves and their classmates. Additionally, they were tough on each other with their evaluation of each other's solutions. One of the best moments in class was when the students in one section were all working on one particularly difficult problem and engaged in a heated debate about how to solve it. They were actually arguing with each other about the mathematics. We thought it was great!

We dedicated the last day of class to a review session based on the TV game show Jeopardy. We downloaded a Jeopardy PowerPoint template from the web [2] and then wrote questions in the following categories: "It's a Mod Mod World," "Graphs, Trees and Paths, Oh My!," "How do I prove thee…," "Let me count the ways," and "Let's talk about sets." We had the students break into three- to four-person teams. Each team had to decide whether or not to wager for each problem. This served as a good method for keeping them all engaged the entire game. Throughout the game the students were diving into their textbooks to remind themselves of the concepts they had forgotten. This served as a good jumpstart for their final exam studying.

By choice, we focused on a team-teaching approach to the course delivery, an approach that continued to evolve throughout the semester. Initially, the course was actually less "team" oriented and more traditional with occasional switching of the instructors between sections. There are a couple of reasons for this. First, each instructor was attempting to develop a bond with his/her section during the initial set of lessons. Secondly, Kevin used this period to sit-in on Rachelle's classes since he was less familiar with presenting the early topics in the course. Additionally, we first met each other at the end of the previous semester and thus had to spend some time getting to know each other both personally and as educators and colleagues. Toward the middle of the course, switching increased significantly. We settled into switching sections for blocks of material instead of individual lessons. Switching lead instructors between blocks increased topic continuity. Finally, toward the end of the course, Kevin took the lead for both sections with Rachelle sitting in and interacting often for both sections. This provided a very dynamic environment in which the students were able to receive a consistent teaching and grading approach, but have the benefit of two instructor perspectives. In addition, both instructors found it quite fun. We tended to feed off each other and bounce

ideas off each other during class, keeping things quite lively.

4. Lessons Learned

This was a tremendous learning experience for both instructors. Not only was it the first time either of us had taught Discrete Math, it was the first time that either of us had team-taught. In addition, it was the first time Kevin had taught a math course. Accordingly, we continually reflected on our delivery while at the same time soliciting feedback from our students.

From our observations, we recognized some benefits from having a CS professor co-teach the course. First, since Kevin is a CS professor he provided direct input into which topics needed to be emphasized to best support the cadets' CS curriculum. In addition, we exploited opportunities to connect with the concepts the cadets were learning in the Data Structures course that they were taking concurrently. We also continuously pointed out where topics they were learning in Discrete Math would be used in their CS studies at West Point.

Having the two professors in the room at the same time provided an invaluable benefit especially since both were new to the course. The professor not in the primary teaching role gave insight on what topics students were having trouble grasping in the lesson. Furthermore, we have two very different, but effective teaching styles. Each of our strengths combined for a stronger course. In particular, Kevin, as a computer science, was able to tie in the various discrete math topics to their computer science studies. While Rachelle, as a mathematician, designed the course structure to ensure that we had a solid, rigorous foundation for mathematical reasoning.

As discussed earlier, we often solicited feedback from our students to keep a pulse on how the course was progressing. From these informal surveys, we found several aspects of the course that the cadets identified as helpful to their learning. The students agreed that the "quick questions" aided them in their self-evaluation of their learning. First and perhaps obviously on the team approach, with two instructors, they had two different viewpoints and two different explanations. Also, having both professors in class kept things interesting and fresh from the students' perspective. The dual presence in class also provided more feedback to students, especially when students were at the board working problems. Finally, having two instructors provided more options for office hours.

However, there were some things that students found distracting. Since we have different teaching styles, when we switched out classes, students found the lessons disjointed at times. In addition, it seemed we had different grading styles. To overcome this, we alternated grading the homework assignments for course consistency. So although we might have different grading styles, with only one professor grading an assignment, all grades were consistent.

We also made some unexpected discoveries. First, adding a computer scientist as a professor did not completely legitimize the necessity of math for the students. Although the vast majority of the students eventually accepted this view, there was a small percentage at the end of the semester who did not accept the need for math in a CS curriculum. Also, Rachelle was surprised with the lack of computer science applications. Instead, Kevin wanted to focus the course material on the 'math' while relying on class discussions for the connections to computer science. Finally, Rachelle found that as the *math* person in the room, she felt sometimes a bit of an outsider.

We constantly had our students working on homework. Each time they handed in an assignment, a new one was handed out. Our goal was to emphasize that the problems were not ones that could be quickly solved in one sitting, but frequently would need to be started, thought about, put away and returned to later. We realized after the first couple of homeworks however that our choice of problems did not reflect this philosophy. We were assigning enough of the easier, recall types of problems that the students could achieve a fairly good grade without actually doing the harder, analysis and synthesis types of problems. Once we realized this, we began assigning mostly the more difficult and challenging problems to make sure that the students really had to wrestle with the material at a higher level. Though we did endure some grumbling from the students, we found that by the end of the semester their work had greatly improved and they were capable of tackling higher-level thinking problems. This was what we had set out to do!

5. Future Undertakings

We hope to teach this course together in the future. Accordingly, we plan to apply the lessons learned to improve the course. In particular, we plan to include more CS applications to aid in the motivation of the material. We will aim to discuss a particular computer science application for each block of material. Next we would like to increase our coordination with the concurrently taught Data Structures course. Although we achieved a limited level of coordination hap-hazardly, we would like to plan inter-course coordination more deliberately in the next iteration. Next, we will consider adding a course-long project or a set of mini-projects. We want to give students more opportunities to synthesize the various topics covered during class. Logistically, we will better plan our switching of *lead* instructor. We will only do these switches between blocks of material. We feel this will help reduce the disjointedness that students sometimes complained about. Finally, we plan on coordinating our grading from the start to provide more consistent feedback for students. We will maintain our high expectations for

the students' mathematical growth by continuing to assign them challenging homework problems and asking them to engage with the course material at a deeper level both in and out of class.

6. Conclusion

Although this was our first time teaching Discrete Math, we feel that the course was highly beneficial for both the students and the professors. The students were challenged daily in a very dynamic environment to improve their mathematical reasoning. We grew tremendously from this experience, both with the course content and delivery. As a by-product of trying so many new things in a single semester, we were probably more reflective during the semester than normal. However, we see this as a win-win for the students and professors since it allowed us to make changes to the course as the semester progressed. We are confident that the lessons we learned last semester will not only help improve our teaching when we teach Discrete Math again, but will improve all our future courses.

Throughout the semester we maintained a course website to which we posted all board problem sheets, homework assignments and exams as well as the solutions to each. The reader is encouraged to visit the site to find these items as well as the Jeopardy game mentioned above. The address of that site is at [3].

References

[1] DeCoste, R. and Huggins, K. Teaching Discrete Math for the First Time. *ACM inroads - SIGCSE Bulletin*. Vol 38, No. 4. ACM Press. New York, USA. December 2006.
[2] http://teach.fcps.net/trt10/PowerPoint.htm
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